INDIAN STATISTICAL INSTITUTE, BANGALORE CENTRE
B.MATH - Second Year, Second Semester, 2015-16
Statistics - II, Midterm Examination, February 25, 2016
Answer all questions.
You may use any result stated in the class by stating it.

1. For observations $Y_1, \ldots, Y_n$, consider the linear model

$$Y_i = \beta_0 + \beta_1 x_i + \epsilon_i, \quad i = 1, \ldots, n,$$

where x_i is the value of a co-variate corresponding to Y_i and ϵ_i are i.i.d. errors having the $N(0, \sigma^2)$ distribution. Here β_0, β_1 and $\sigma^2 > 0$ are unknown parameters and x_i are treated as known constants. Also $-\infty < \beta_0 < \infty$, $-\infty < \beta_1 < \infty$.
(a) Show that the distribution of $Y_1, \ldots, Y_n$ belongs to k-variate exponential family. Find k.
(b) Use properties of exponential family to set up equations and solve them to find MLE of $(\beta_0, \beta_1, \sigma^2)$. [12]

2. Consider a family of regular models with density $f(x|\theta)$ such that $f(x|\theta) > 0$ for all $\theta \in \Theta$ and for all $x \in \mathcal{X}$. Suppose $T(X)$ is sufficient for this family of distributions indexed by θ. Show that if $T(x) = T(y)$ for two sample points x and y then $\frac{f(x|\theta)}{f(y|\theta)}$ is free of θ. [6]

3. Roll a balanced six-faced die and let N denote the number of dots that show up. Having observed $N = n$, perform n Bernoulli(θ) trials, getting X successes.
(a) Find a minimal sufficient statistic for θ.
(b) Show that minimal sufficient statistic is not complete in this case. [7]

4. Suppose $X_1, X_2, \ldots, X_n$ are i.i.d. observations from Exponential(λ) (with density proportional to $\exp(-\lambda x)$), where $n \geq 3$ and $0 < \lambda$.
(a) Does this belong to exponential family of distributions? Justify.
(b) Find the UMVUE of λ.
(c) Does the UMVUE attain the C-R lower bound? [15]

5. Consider a random sample $X_1, X_2, \ldots, X_n$ from $U(0, \theta)$, where $\theta > 0$.
(a) Construct a 95% confidence interval for θ which has the form:
$[X_{(n)}, X_{(n)}/c]$ for some constant c.
(b) If the confidence interval constructed from observed data is the interval $[1, 12.5]$, how will you interpret it? [10]